Stage 3 ★
Mixed Selection 1 – Solutions

1. Street lamps

Note that there are 9 gaps between the lamps, so the total time is (5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13) seconds = 81 seconds.

2. Printing error

After every two numbers, one is omitted. Because $89 = 2 \times 44 + 1$, there must be 44 page numbers missing and so the number on the last page is $89 + 44 = 133$.

3. Luis' Seven

If the smallest number that Luis writes down is n, then the numbers in order are: n, $n + 1$, $n + 2$, $n + 3$, $n + 4$, $n + 5$ and $n + 6$.

The sum of the three smallest numbers is therefore

$$n + (n + 1) + (n + 2) = 3n + 3.$$

As the three smallest numbers total 33, $3n + 3 = 33$.
Subtracting 3 from both sides gives $3n = 30$, and so $n = 10$.

Therefore the numbers are 10, 11, 12, 13, 14, 15 and 16, so the total of the three largest numbers is 14+15+16=45.

4. Mean sequence

Suppose the first two terms of the sequence are x and y. The third term is $\frac{1}{2}(x + y)$, and the fourth term is $\frac{1}{4}(x + 3y)$ so the fifth term is $\frac{1}{8}(3x + 5y)$. Putting $x = \frac{2}{3}$ and $y = \frac{4}{5}$ we obtain $\frac{1}{8}(2 + 4) = \frac{3}{4}$.
Alternatively, we can just write out the sequence by repeatedly taking means
The first term is $\frac{2}{3}$ and the second term is $\frac{4}{5}$.
So the third term is $\frac{1}{2}\left(\frac{2}{3} + \frac{4}{5}\right) = \frac{1}{2}\left(\frac{10+12}{15}\right) = \frac{1}{2}\left(\frac{22}{15}\right) = \frac{11}{15}$
and the fourth term is $\frac{1}{2}\left(\frac{4}{5} + \frac{11}{15}\right) = \frac{1}{2}\left(\frac{12+11}{15}\right) = \frac{1}{2}\left(\frac{23}{15}\right) = \frac{23}{30}$
and, finally, the fifth term is $\frac{1}{2}\left(\frac{11}{15} + \frac{23}{30}\right) = \frac{1}{2}\left(\frac{22+23}{30}\right) = \frac{1}{2}\left(\frac{45}{30}\right) = \frac{1}{2}\left(\frac{3}{2}\right) = \frac{3}{4}$

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk).

5. Expanding pattern

One way to proceed is to regard the pattern as four arms, each two squares wide, with four corner pieces of three squares each. So for the nth pattern, we have $4 \times 2 \times n + 4 \times 3 = 8n + 12$ (there are other ways to achieve this formula). For $n = 10$, we need $8 \times 10 + 12$, i.e. 92 squares. *Alternatively*, you can draw the pattern for $n = 10$ and just count the squares in a systematic way.

6. Fruit line-up

Each fruit can have at most two fruits next to it but each type of fruit must be next to three other types of fruit so there are at least two of every fruit. This means there are at least 8 fruits in total. In fact 8 are sufficient, as shown in the arrangement

OABPOBAP (O for Orange, P for Peach, A for Apple, B for Banana)